MA861 A1 Course Syllabus

Numerical Solution of Partial Differential Equations
T/TH 9:30-10:45AM, PSY B37

Instructor Information:

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Professor
Department of Mathematics and Statistics
CDS 442

Course Webpage

We will use the course Blackboard page for posting/submitting all assignments and material.

Office Hours:

In my office, CDS 442. Time TBA based on final registration and everyone's schedules.

Description and Goals:

This course provides an introduction to the numerical solution and approximation of partial differential equations. The course will balance theoretical topics, such as consistency, stability, well-posedness and convergence, with practical considerations, such as algorithmic complexity, fast solution methods, and the development/implementation of numerical solvers. It will function as a survey of major classical methods (i.e. finite difference, finite volume, finite element, and spectral methods), while also mentioning some more recent approaches (neural network approximation methods).

Tentative Course Outline:

This is probably too much material, but we will adjust topics based on everyone's interests (i.e. more breadth of methods vs. more depth on specific methods / fast solution approaches):

Finite difference methods for the Advection equation
- – Difference operators, and implicit vs. explicit methods.
- – Boundary conditions, matrix representations for implicit methods, computational work to solve associated linear systems.
- – Consistency, wellposedness, stability, CFL conditions and convergence on the real line.

Finite difference methods for the Heat and Wave equations
- – 1D Laplacian and Method of Lines.
- – Finite volume methods and structure-preserving discretizations.
- – Fourier Series.
- – General results on consistency, wellposedness, stability, CFL conditions and convergence on the real line.
- – Boundary conditions.
- – Discontinuities and numerical approximation to Green's functions.

2D Poisson Equation and Elliptic Equations
- – Boundary conditions and properties of 2D discrete Laplacian; solvability and maximum principle.
- – Fast numerical solution methods; linear algebra approaches
  - – Direct vs. sparse vs. banded solvers.
  - – Iterative methods: classical methods and Krylov space methods. Preconditioning.
  - – Multigrid
  - – Fast Fourier Transform (FFT) approaches
- – finite element methods; convergence and implementation on general smooth 2D domains.

2D/3D Heat and Wave equations
- – finite difference and finite element methods, computational considerations in higher dimensions.

Spectral Methods for 1D Evolutionary Equations
Neural Network Methods (DGM / PINNs)
Select topics for nonlinear equations
- – Reaction-diffusion equations; temporal discretization approaches
  - – Implicit methods, IMEX methods, Splitting methods, and Exponential Integrators
  - – Jacobian approximation approaches: finite difference vs. automatic differentiation.

Prerequisites:

Undergraduate level linear algebra, ODEs and PDEs. Programming experience in one of Julia, Matlab, or Python. Undergraduate complex variables and (undergrad) real analysis recommended.

Textbooks:
As I don't think there is any one "best" numerical PDE book, I will pull material from several, trying to stick to ones you can freely obtain digitally via the library or web. I'll definitely reference material from the following, for which only the Johnson book is not available from the library online (but it is a very cheap Dover book, so I'd suggest just purchasing it).

- Finite Difference Schemes and Partial Differential Equations, Second Edition, John C. Strikwerda, SIAM
  - Note, this book is available to download for free through the BU library: here.
- Applied Numerical Linear Algebra, James W. Demmel, SIAM
  - Note, this book is available to download for free through the BU library: here.
- Numerical Solution of Partial Differential Equations by the Finite Element Method, Claes Johnson, Dover.
- Chebyshev and Fourier Spectral Methods, JP Boyd: here.

I suggest downloading the books right at the start of the semester, as sometimes the library loses access as the semester progresses for short periods of time.

The following is probably my favorite book for this class, which I only very recently discovered, and is perhaps the most comprehensive in covering all the class material. Unfortunately it is from Wiley, so the library does not have free access to it (and it is quite expensive). However, it will probably be the closest single book to covering all the class notes:

- Numerical Analysis of Partial Differential Equations, S. H. Lui, Wiley (2011).

Supplemental Textbooks:
You may find the following useful as references / supplements:

A broad summary-type introduction that is easier to read but more high level than the above books is
- Finite Difference Methods for Ordinary and Partial Differential Equations, LeVeque, SIAM
  - Note, this book is available to download for free through the BU library: here.

Some other introductory books that you might prefer include
- A First Course in the Numerical Analysis of Differential Equations, Arieh Iserles, Cambridge.
- Numerical Partial Differential Equations; Finite Difference Methods, J. W. Thomas, Springer.
- Numerical Partial Differential Equations; Conservation Laws and Elliptic Equations, J. W. Thomas, Springer.
- Time Dependent Problems and Difference Methods, Gustafsson, Kreiss and Oliger, Wiley.

An alternative to Demmel that some find easier to read, but covers a lot of the same material, is
- Numerical Linear Algebra by Trefethen and Bau
  - Note, this book is available to download for free through the BU library: here.

Requirements and Grading:
70% - Around five homework assignments involving both analytical calculations and implementation / use of numerical methods.
30% - Final project presentation and report.

Possible Project Topics:
- Boundary Element Methods
- Navier Stokes.
- Immersed Boundary Method.
- Modern numerical libraries (PETSC, libmesh, Fenics, Gridap, etc).
- Discontinuous Galerkin methods.
- Hyperbolic systems.
- Surface PDEs.
- Adaptive mesh refinement.
- Specific nonlinear equations.
- Fast multipole method.

- HP spectral methods.
- PDE-specific temporal discretization methods (SSP methods, Runge-Kutta Chebyshev methods).
- Neural network methods for high dimensional PDEs.

Homework Policy:
Homework assignments will require a mix of analytical calculations and numerical simulations and should be submitted as pdf reports on Gradescope. Formatting instructions for submission will be provided in each assignment.

Programming Languages
You are free to use Julia, Matlab, or Python for the course assignments. I can help with all three, though my Python knowledge is much less than the other two (so I may not be able to help with issues as quickly / effectively). Any code I illustrate in class and post will be in Julia, along with reference code for assignments.